



ABOUT THE DYNAMICS OF HETEROGENEOUS FINE-GRAINED MIXTURE ON A FLAT VIBROSIEVE

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Abstract

With the use of two-parameter rheological dependence, the reserved formulas shown out for the calculation of speed of grain stream and productivity of vibrosieve on tails faction. As well as in other continual models, tangent tension in mixture, with its shear motion is accepted proportional to the velocity shear strain at its motion of change, but additionally also taken into account the residual friction. It is set proportional internal overpressure in the mixture. Heterogeneity of fine-grained mixture on the thickness of mobile layer characterizes variables by porosity (or by specific mass) and dynamic coefficient of vibroviscosity. Distributing of concentration of grains, whether to porosity, on the thickness of separated layer it is accepted dependency upon the type of speed of grain stream, so that, where a most rate of movement is, most porosity answers a transversal co-ordinate, and to the least – the least. For the variant of linear change of coefficient of vibroviscosity pseudo rarefied grain mixture on the height of its layer worked out a differential equation of the second order after variable coefficients in relation to the rate of movement. In the reserved form, it is built analytical decision, which satisfies the given boundary conditions. In a result, speed of grain stream and productivity of vibrosieve after stair faction is expressed through the modified Bessel functions. Therefore, for the leadthrough of engineering calculations, it is possible to use these special function tables. It is shown that from the removed formulas, as a result of limit transition as separate cases, come up the known dependences received earlier for calculation of the movement of grain mixture with linearly variable coefficient of vibroviscosity. Carrying out calculations constructed profiles of speed of a grain stream. They little differ from rectilinear for the chosen option of change of coefficient of vibroviscosity. On numerical examples, influence of values of rheological constants on distribution of porosity and on kinematic characteristics of a grain stream is analyzed. It is shown that settlement results significantly depend on values of constants of model.

Key words: flat inclined vibrosieve, settled grain stream, heterogeneous mixture, variable porosity, variable coefficient of vibroviscosity, rate of movement, productivity of vibrosieve.

RESEARCH ANALYSIS

At high speeds of motion of mixture on the sieve of complete length, without regard to megascopic porosity, sifted faction does not have time fully to move away from stair faction. This situation arises up at the small rates of movement at small porosity, because the process of segregation is slowed. It is therefore desirable

to have such the speed and porosity, that the complete division of factions was arrived at the proper productivity of vibrosieve. It is for this purpose needed to know influence of different factors on the rate of movement and distributing of porosity, that possibly at presence of adequate mathematical models, that is why developments of them behave to the actual tasks.

Known researches of motion of grain mixture of variable porosity on a flat vibrosieve based on the numerical methods of calculation ((Tishchenko *et. al* 2002; Piven 2006; Tishchenko *et. al* 2007; Tishchenko *et. al* 2015). At first integrate the special nonlinear differential equalization of the second order the there adopted methods and determine distributing of concentration of grains on the height of separated layer. Then got results take into account at numerical integration of differential equalization of corn stream, to find the rate of movement and other kinematics descriptions. With insignificant changes this approach is used for the calculations of grain streams in the vertical cylindrical sieves of vibrocentrifuges (Tishchenko *et. al* 2003; Tishchenko *et. al* 2010; Tishchenko *et. al* 2013; Tishchenko *et. al* 2016), where additionally take into account tasks do for workers (Olshanskyi *et. al* 2016; Olshanskyi *et. al* 2016; Olshanskii *et. al* 2016), by introduction of analytical approximation of distribution of porosity on the thickness of movable layer. Common fault of the called works is that in them dependence of distribution of porosity of grain mix on the speed of its movement is not considered. Authors of the monograph pay attention to it (Dolgunin *et. al* 2005).

Coming from the examined researches, the mathematical model of motion of fine-grained mixture on a flat vibrosieve, in that the change of porosity and change of coefficient of vibroviscosity are simultaneously taken into account on the thickness of separated layer, is here offered. According to the accepted dependence, in overhead part of layer, near his

free surface, specific mass of mixture less than in an underbody that contacts with the surface of sieve. Just the same distribution is peculiar and rates of movement. Therefore high speed of motion is answered by less specific mass and vice versa. The coefficient of vibroviscosity of the pseudorarefied grain mixture is accepted too to the variables on the height of movable layer. It is the largest at the bottom of the layer near the surface of vibrosieve and smallest at the top, near the free surface of the mixture. So we are talking about moving of the pseudorarefied heterogeneous mixture both after a closeness and after vibroviscosity.

THE AIM OF THE WORK

The purpose of the article is a leadingout and approbation the calculations of formulas for the calculation of kinematics descriptions of grain stream and productivity of vibrosieve, when porosity of mixture and coefficient of its vibroviscosity change on the height of mobile layer.

RESULTS AND DISCUSSION

Use a calculation chart, represented on a Fig. 1. Here h – thickness of mobile layer of mixture; θ – angle of slope of sieve is to horizon; A^* – amplitude of longitudinal vibrations of sieve is with frequency ω ; x, y – accordingly longitudinal and transversal coordinates; $u = u(y)$ – a rate of a withstand movement of mixture is in direction of axis ox .

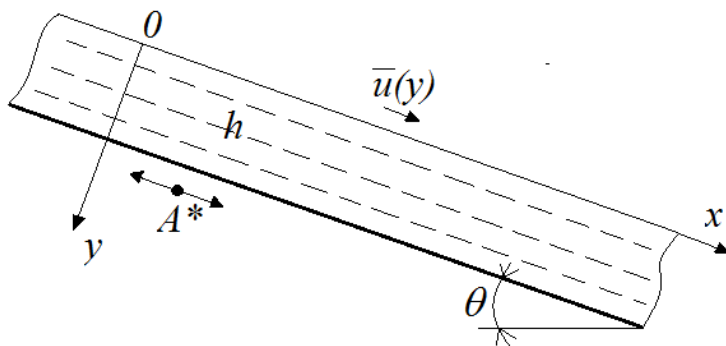


Figure 1. Calculation chart

Differential equalization of grain-growing stream of heterogeneous mixture is shown out after variables by specific mass and coefficient of vibroviscosity as known tangent tension $\tau = \tau(y)$ in mixture, that carries out motion of change, satisfies to equalization (Tishchenko *et. al* 2011):

$$\frac{d\tau}{dy} = -\gamma g \sin \theta v(y). \quad (1)$$

Here γ – specific mass of material of grains is in mixture; g – acceleration of the free falling; $v(y)$ – concentration of grains is on the thickness of movable layer of mixture.

Product describes the change of specific mass of heterogeneous mixture on transversal coordinates y .

Connection between tangent tension and rate of movement of $u = u(y)$ express two-parameter rheological dependence:

$$\tau = \mu(y) \frac{du}{dy} + fp(y) \operatorname{sign} \left(\frac{du}{dy} \right), \quad (2)$$

where $\mu(y)$ – variable on the thickness of layer dynamic coefficient of vibroviscosity of mixture; f – coefficient of remaining internal dry friction; $p(y)$ – surplus intrinsic pressure.

Taking into account, that $\frac{du}{dy} \leq 0$, $\frac{dp}{dy} = \gamma g \cos \theta v(y)$, after a substitution (2) in (1), get:

$$\frac{d\mu}{dy} \frac{du}{dy} + \mu \frac{d^2 u}{dy^2} = -\gamma g (\sin \theta - f \cos \theta) v(y). \quad (3)$$

Take the function of $v(y)$ in a kind (Tischenko *et. al* 2011):

$$v(y) = v_* \frac{1 - \delta u(y)}{1 - \delta u_*}, \quad (4)$$

where $1 - \delta u(0) > 0$; v_* – concentration of grains is in mixture near the surface of sieve ($y = h$); δ – rheological constant, that depended on the presence of activators of process of segregation on the surface of sieve; $u_* = u(h)$ – speed of sliding the mixture on surface of sieve.

Equalization (3), after a substitution for him (4), a kind acquires:

$$\frac{d^2 u}{dy^2} + \frac{1}{\mu} \frac{d\mu}{dy} \frac{du}{dy} - \frac{b\delta}{\mu} u = -\frac{b}{\mu}, \quad (5)$$

$$\text{thus } b = \frac{\gamma g v_*(\sin \theta - f \cos \theta)}{1 - \delta u_*}; \sin \theta - f \cos \theta > 0.$$

For a specification, will consider a variant farther, when the dynamic coefficient of vibroviscosity depends linear character on a coordinate y , that is formula takes place (Tischenko *et. al* 2011; Tischenko 2012):

$$\mu(y) = ay,$$

where a -const.

In this case (5) passes to heterogeneous differential equalization by Bessel:

$$\frac{d^2 u}{dy^2} + \frac{1}{y} \frac{du}{dy} - \frac{c}{y} u = -\frac{c}{\delta y}, \quad (6)$$

in that $c = b\delta a^{-1} > 0$.

By regional terms to (6) take expressions:

$$\mu(y) \frac{du}{dy} \Big|_{y=0} = 0; u(h) = u_*, \quad (7)$$

from that the first expresses absence of tangent tension on free surface of mixture.

The common decision of equalization (6) looks like:

$$u(y) = AI_0(2\sqrt{cy}) + BK_0(2\sqrt{cy}) + \frac{1}{\delta}, \quad (8)$$

where A, B – free constants; $I_0(z), K_0(z)$ – modified Bessel functions and Macdonald function of zero indexes.

Function $K_0(z)$ singular (has a logarithmic feature) in the point of $z = 0$. Therefore, to get a limit speed of $u(y)$, that answers practice of separation accept in (8) $B = 0$. Thus automatically executed and the first condition in (7), because $\mu(0) = 0$. Meeting the second condition in (7), find, that:

$$A = \frac{u_* - 1/\delta}{I_0(2\sqrt{ch})}.$$

Thus, speed of grain-growing stream of heterogeneous mixture is given by expression:

$$u(y) = \frac{1}{\delta} + \left(u_* - \frac{1}{\delta} \right) \frac{I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})}, \quad (9)$$

the decision (8) passes to that.

At the calculations of $u(y)$ on the formula (9) it is possible to use special function printed in tables (Abramovits *et. al* 1979; Yanke *et. al* 1977).

A formula (9) is useless for the calculations of motion of mixture of permanent specific mass, when $\delta = 0$, because has a vagueness: $|\infty - \infty|$. To expose this vagueness, give (9) a next kind:

$$u(y) = \frac{u_* I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})} + \frac{1}{\delta} \frac{I_0(2\sqrt{ch}) - I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})}. \quad (10)$$

Will take into account farther, that at $z \ll 1$: $I_0(z) \approx 1 + \frac{z^2}{4}$.

Then:

$$\lim_{\substack{\delta \rightarrow 0 \\ (c \rightarrow 0)}} \frac{I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})} = 1; \quad \lim_{\substack{\delta \rightarrow 0 \\ (c \rightarrow 0)}} \frac{1}{\delta} \frac{I_0(2\sqrt{ch}) - I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})} = \frac{b_*}{a} (h - y);$$

$$b_* = \gamma g v_*(\sin \theta - f \cos \theta).$$

Therefore maximum transition $\delta \rightarrow 0$ ($c \rightarrow 0$) в (10) gives:

$$u(y) = u_* + \frac{b_*}{a} (h - y). \quad (11)$$

Thus, at $\delta = 0$, speed has a rectilinear profile, that is it depends linear character on a coordinate y . If in (11) to put $u_* = 0$; $f=0$ and to designate $a_* = a/(\gamma v_*)$, then it will purchase kind:

$$u(y) = \frac{g \sin \theta}{a_*} (h - y).$$

A just the same formula was before shown out in (Tischenko *et. al* 2011) within the framework of hydrodynamic model. It is answered by the linear profile of speed of grain stream.

From (9) the next formula of high speed of motion hatches:

$$\max u = u(0) = \frac{1}{\delta} + \left(u_* - \frac{1}{\delta} \right) \frac{1}{I_0(2\sqrt{ch})}.$$

In engineering calculations sometimes use the mean value of speed (Tischenko *et. al* 2011):

$$u_m = \frac{1}{h} \int_0^h u(y) dy. \quad (12)$$

In this case it is too expressed through cylindrical functions, because:

$$u_m = \delta + \frac{(u_* - 1/\delta)}{h I_0(2\sqrt{ch})} \int_0^h I_0(2\sqrt{cy}) dy. \quad (13)$$

Will replace in (13) a variable integration of y on t^2 . Then:

$$\int_0^h I_0(2\sqrt{cy}) dy = 2 \int_0^{\sqrt{h}} t I_0(2\sqrt{ct}) dt.$$

As (Gradshtein *et. al* 1962):

$$I_n(z) = i^{-n} J_n(iz); n = \overline{0;1}; i = \sqrt{-1}; \int z J(z) dz = z J_1(z),$$

then:

$$\int_0^h I_0(2\sqrt{cy}) dy = \frac{\sqrt{h}}{\sqrt{c}} I_1(2\sqrt{ch}). \quad (14)$$

Here $I_1(z)$ – the modified Bessel functions is an index unit.

Thus,

$$u_m = \frac{1}{\delta} + \frac{(u_* - 1/\delta) I_1(2\sqrt{ch})}{\sqrt{ch} I_0(2\sqrt{ch})}. \quad (15)$$

A formula (15) loses an action at $\delta = 0$. To expose a vagueness, will give (15) a form:

$$u_m = u_* \frac{I_1(2\sqrt{ch})}{\sqrt{ch} I_0(2\sqrt{ch})} + \frac{1}{\delta} \frac{\sqrt{ch} I_0(2\sqrt{ch}) - I_1(2\sqrt{ch})}{\sqrt{ch} I_0(2\sqrt{ch})}. \quad (16)$$

Will take into account farther, that at $z \ll 1$: $I_1(z) \approx \frac{z}{2} + \frac{1}{2} \left(\frac{z}{2}\right)^3$. Then, maximum passing $\delta \rightarrow 0$ ($c \rightarrow 0$) to (16) get:

$$u_m = u_* + \frac{b_*}{2a} h.$$

Certainly, this formula can be got and simpler by a substitution (11) in (12), but without verification of authenticity (15).

Using u_m , comfortably approximately to determine the productivity of vibrosieve on stair faction of P_m , as:

$$P_m = \frac{\gamma H v_* h}{1 - \delta u_*} u_m (1 - \delta u_m) = \frac{\gamma H v_* \sqrt{h}}{\sqrt{c}} \frac{I_1(2\sqrt{ch})}{I_0(2\sqrt{ch})} \left[\frac{1}{\delta} + \frac{(u_* - 1/\delta) I_1(2\sqrt{ch})}{\sqrt{ch} I_0(2\sqrt{ch})} \right]. \quad (17)$$

Here H – width of the perforated part of sieve.

Maximum transition $\delta \rightarrow 0$ ($c \rightarrow 0$) to (17) gives:

$$P_m = \gamma H v_* h \left(u_m + \frac{b_* h}{2a} \right). \quad (18)$$

At more exact calculation of the productivity of vibrosieve it will be to find an integral:

$$P = \frac{\gamma H v_*}{1 - \delta u_*} \int_0^h u(y) [1 - \delta u(y)] dy.$$

It also expressed through tabulated cylinder functions, because recognition (9):

$$P = \frac{\gamma H v_*}{\delta I_0(2\sqrt{ch})} \int_0^h I_0(2\sqrt{cy}) \left[1 + (\delta u_* - 1) \frac{I_0(2\sqrt{cy})}{I_0(2\sqrt{ch})} \right] dy.$$

Here, except for (14), should additionally to calculate an integral:

$$\int_0^h [I_0(2\sqrt{cy})]^2 dy = 2 \int_0^{\sqrt{h}} t [I_0(2\sqrt{ct})]^2 dt.$$

Given that (Yanke *et. al* 1977):

$$\int z J_0^2(z) dz = \frac{z^2}{2} [J_0^2(z) - J_{-1}(z) J_1(z)]; \quad J_{-1}(z) = J_1(z),$$

get:

$$\int_0^h [I_0(2\sqrt{cy})]^2 dy = h [I_0^2(2\sqrt{ch}) - I_1^2(2\sqrt{ch})].$$

Thus:

$$P = \frac{\gamma H v_* \sqrt{h}}{\delta I_0(2\sqrt{ch}) \sqrt{c}} \left\{ I_1(2\sqrt{ch}) + \frac{\sqrt{ch}(\delta u_* - 1)}{I_0(2\sqrt{ch})} [I_0^2(2\sqrt{ch}) - I_1^2(2\sqrt{ch})] \right\}. \quad (19)$$

The maximum passing $\delta \rightarrow 0$ ($c \rightarrow 0$) to the formula (19) too results in (18).

As specified higher, at the calculation of speed of grain stream and productivity of vibrosieve of value $I_0(z)$ i $I_1(z)$ it comfortably to find by means of special function tables or on a computer (Dyakonov 2003). But in practice of separation calculation parameters are such, that the arguments of the indicated functions are small that is why they can be calculated and after formulas:

$$I_0(z) = 1 + \left(\frac{z}{2}\right)^2 + \frac{1}{4}\left(\frac{z}{2}\right)^4 + \frac{1}{36}\left(\frac{z}{2}\right)^6 + \frac{1}{576}\left(\frac{z}{2}\right)^8 + \dots$$

$$I_1(z) = \frac{z}{2} + \frac{1}{2}\left(\frac{z}{2}\right)^3 + \frac{1}{12}\left(\frac{z}{2}\right)^5 + \frac{1}{144}\left(\frac{z}{2}\right)^7 + \frac{1}{2880}\left(\frac{z}{2}\right)^9 + \dots$$

if $z \leq 2$.

For realization of calculations accept: $\gamma = 1350 \text{ kg/m}^3$; $h = 0,012 \text{ m}$; $\theta = 8^\circ$; $u_* = 0,04 \text{ m/s}$ and different values: a , δ , v_* , f .

Bring a calculation over of porosity $\varepsilon(y)$ of grain mixture on a formula:

$$\varepsilon(y) = 1 - v(y).$$

Results of calculations $u(y)$ and $\varepsilon(y)$, at $a = 30 \text{ Pa}\cdot\text{s/m}$; $\delta = 2 \text{ s/m}$; $f = 0,07$ and three values of v_* , as charts, it is represented on Fig. 2 and Fig. 3.

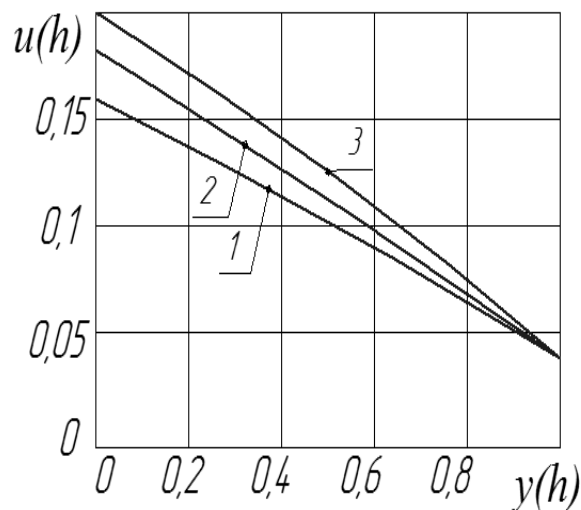


Figure 2. Profiles of velocity of grain flow with different v_* :
1 - $v_* = 0,4$; 2 - $v_* = 0,5$; 3 - $v_* = 0,6$

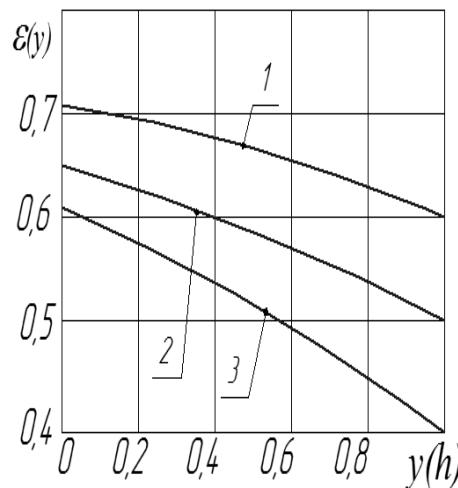


Figure 3. Distribution of porosity along the height of the moving layer of the mixture: 1 - $v_*=0,4$; 2 - $v_*=0,5$; 3 - $v_*=0,6$

Graphs are protuberant, but have small curvature. Most porosity takes place near free surface of mixture $y = 0$. The most rate of movement is there arrived at.

Fig. 4 and Fig. 5 shows graphs $u(y)$ and $\varepsilon(y)$, calculated for two values δ . To get it set: $a=30 \text{ Pa} \cdot \text{s/m}$; $v_*=0,5$; $f = 0,07$.

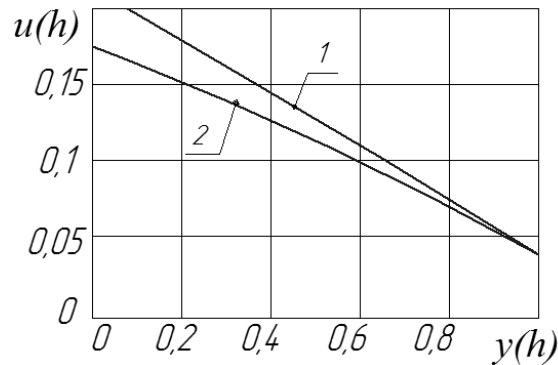


Figure 4. Profiles of velocity of grain flow with different δ :
1 - $\delta = 0,75$; 2 - $\delta = 2,25$

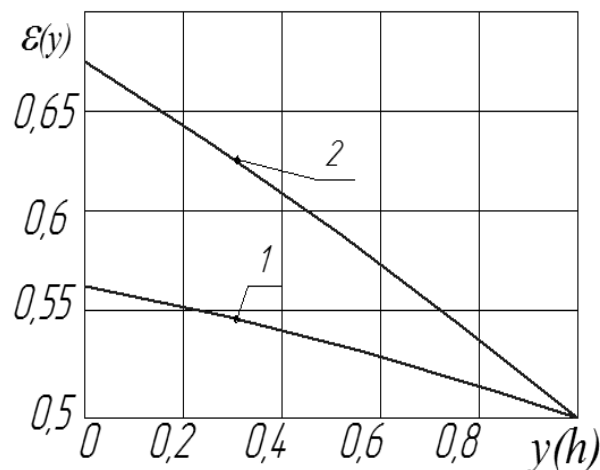


Figure 5. Distribution of porosity along the height of the moving layer of the mixture: 1 - $\delta = 0,75$; 2 - $\delta = 2,25$

As see, value δ substantially influences on changing of porosity on the thickness of mobile layer, while its influence on the rate of movement is unimportant.

High and middle speeds of grain stream also depends on the values of rheological constants a and f . It is confirmed by the results of calculations $u(0)$ and u_m , written in a table 1. For the leadthrough of calculations, except for the indicated higher numeric data, set additionally; $V_*=0,6$; $\delta = 2 \text{ s/m}$.

Table 1. High (numerators) and middle (denominators) speeds of grain stream at different a, f

a , $\text{Pa} \cdot \text{s/m}$	$f = 0$	$f = 0,03$	$f = 0,06$	$f = 0,09$
	Value 10 $u(0)$ and 10 u_m , m/s			
20	3,49	3,16	2,72	2,11
	2,11	1,90	1,63	1,29
30	2,93	2,59	2,18	1,66
	1,76	1,56	1,33	1,05

40	$\frac{2,53}{1,53}$	$\frac{2,22}{1,35}$	$\frac{1,85}{1,15}$	$\frac{1,40}{0,91}$
50	$\frac{2,24}{1,36}$	$\frac{1,95}{1,20}$	$\frac{1,62}{1,02}$	$\frac{1,22}{0,82}$

Calculated for the same information of the productivity of vibrosieve written in a table 2. Values in numerators obtained on a formula (19), and in denominators – on a formula (17). As see, a formula (17) gives the overpriced results.

Table 2. The productivity vibrosieves, calculated on formulas (19) – numerators and (17) – denominators

a , Pa·s/m	$f = 0$	$f = 0,03$	$f = 0,06$	$f = 0,09$
	Value P/H and P_m/H , kg/ (m s)			
20	$\frac{1,122}{1,289}$	$\frac{1,111}{1,244}$	$\frac{1,067}{1,162}$	$\frac{0,959}{1,011}$
30	$\frac{1,092}{1,204}$	$\frac{1,050}{1,135}$	$\frac{0,975}{1,031}$	$\frac{0,847}{0,875}$
40	$\frac{1,040}{1,120}$	$\frac{0,983}{1,041}$	$\frac{0,897}{0,934}$	$\frac{0,768}{0,786}$
50	$\frac{0,987}{1,047}$	$\frac{0,923}{0,965}$	$\frac{0,834}{0,860}$	$\frac{0,712}{0,724}$

CONCLUSIONS

Unlike existent theories, offered here mathematical model of motion of heterogeneous mixture from the rate of its movement and change of vibroviscosity after a linear law on the thickness of mobile layer of friable material. The shown out formulas enable to calculate speed of grain stream and productivity of vibrosieve on stair faction using tables by modified Bessel functions or using a computer. It is set during calculations, that on the values of rheological constants of model theoretical results depend substantially, that their accordance practice of separation. Therefore, for the concordance of theory with an experiment, need the proper authentication of model constants.

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