



RESEARCH OF DYNAMICS OF HYDRAULIC DRIVE WITH HYDRAULIC DISTRIBUTOR

Loveikin V. S.^a, Spodoba O. O.^b, Spodoba M. O.^c, Romasevych Yu. O.^d

National University of Life and Environmental Sciences of Ukraine, Heroiv Oborony Street, 15, Kyiv, 03041, Ukraine

Corresponding author's e-mail: lovvs@ukr.net^a, sp1309@ukr.net^b, spmisha@ukr.net^c,
romasevichyuriy@ukr.net^d

Abstract

The dynamic model of hydraulic drive of double-acting is considered in the work. The model consists of a hydraulic cylinder, a hydraulic distributor, a hydraulic pump and hydraulic pipelines. The mathematical model in the form of a single-mass dynamic system, a hydraulic drive of double-acting is constructed. The mathematical model is constructed taking into account the compressibility of the working fluid, the rigidity of the elements of the hydraulic system and the forces of viscous and semi-dry friction. The hydraulic distributor is considered as the control equipment of the double-acting hydraulic cylinder. It redistributes the flow of the working fluid in the hydraulic system, as well as changing the speed of movement of the piston rod of the hydraulic cylinder. Moving the valve spool relative to the housing forms a cross-section between the edges of the valve spool and the inlet and outlet ducts of the housing. The parameters of the influence of the dependence of the change in the area of the cross-section formed between the edges of the valve spool and the inlet and outlet channels of the housing during the mode of start, acceleration and exit to steady motion are analysed. Their significant influence on the occurrence of dynamic loads during the transitional period in the hydraulic system is revealed. Dynamic loads, in turn, adversely affect the hydraulic system elements and, accordingly, the mode of movement of the cylinder piston rod. The result of this work is the solution of a mathematical model of a dynamic hydraulic drive system. The simulated results are presented graphically for ease of analysis.

Key words:

hydraulic cylinder, dynamic model, mathematical modelling, dynamic loads, hydraulic drive.

Introduction

The hydraulic drive is widely used in hoisting-and-transport, agricultural, forestry, municipal mining and other types of equipment, as a drive of working bodies. The most commonly used hydraulic drive with throttle control. The quality of the hydraulic system during transient motion depends on the dynamic loads that arise during operation. Therefore, in theoretical studies of volumetric hydraulic systems, the development of adequate mathematical models taking into account the geometric parameters of hydraulic control equipment are an important factor.

Formulation of problem

In most cases, the hydraulic drive control is performed using a hydraulic distributor.

The hydraulic distributor is the most critical element of the hydraulic drive. Its task is not only to redistribute the flow of the working fluid in the system but also to change the speed of the piston rod of the hydraulic cylinder (Bashta, 1971).

The hydraulic drive control process is based on the principle of changing the volumetric flow of the working fluid using a hydraulic distributor. In this case, the main energy parameters of the hydraulic drive are the pressure and volumetric flow rate of the working fluid, which determine the power of the hydraulic drive (Zezin *et al.* 2011).

The efficiency of using the hydraulic drive power

will depend on the control method and the method of connecting the hydraulic distributor (Popov, 1987). The efficiency of the hydraulic drive as a whole is expressed in the exact positioning of the actuating device. In order to achieve positive results, a positional feedback hydraulic actuator is used (Zhdanov, 2016; Mints, 2012). The proportional-integral-differential control is also widely used to control the electro-hydraulic servo system, but the use of this approach is limited due to the nonlinear dynamics characterizing these systems (Ming *et al.* 2014). Adaptive feedback systems are also used (Detiček, 2011).

Dynamic processes in a hydraulic drive, as a rule, are characterized by fluid compressibility, pressure fluctuations and the nature of the load (Mishuk, 2016). Also, internal dynamic processes in the distributor itself can have a negative impact on the dynamics of the hydraulic system as a whole (Zhilevich *et al.* 2013).

With a large amount of consideration of the problem of mathematical modelling and dynamic analysis of the hydraulic drive, the solution to this problem is an urgent task.

Purpose of research

The purpose of these studies is to build a mathematical model of the hydraulic drive and to study the dynamic loads in the hydraulic system in the transitional period of motion under different laws of change of the area of the cross-section formed between the edges of the slide valve and the inlet and outlet ducts of the housing.

Research results and discussion

The mathematical model of the double-acting hydraulic cylinder is constructed in the form of a single-mass dynamic system, which is the simplest in the study of the dynamic loads arising in the hydraulic system in the transitional period of motion. When constructing a mathematical model of the hydraulic drive process (Fig. 1) and performed dynamic analysis, we accept the following assumptions:

- the hydraulic cylinder is arranged vertically. As this arrangement is inherent in most lifting machines and mechanisms (Pelevin *et al.* 2015.).
- the temperature, viscosity of the working fluid and the amount of undissolved air in it remain unchanged;
- the masses of the output link, the load and the working fluid in the hydraulic cylinder are concentrated in the geometric centres of mass and reduced to the piston;
- the working fluid is considered with the condition of compressibility;
- structural elements of the hydraulic system, with the exception of pipelines are considered absolutely rigid elements. That is, at elevated pressure radial deformation perceives the pipeline;

- wave phenomena occurring in pipelines do not take into account, considering that the frequency of wave processes exceeds the oscillation frequency of the hydraulic system by an order of magnitude (Anisimov *et al.* 2012).

Based on the design scheme (Fig. 1) on the hydraulic cylinder during its movement will act:

- the forces of inertia resulting from the displacement of the masses of the moving parts reduced to the piston of the hydraulic cylinder, which move at the appropriate speed and acceleration;
- forces of semi-dry friction in seals and guide bushings of the piston rod and piston of the hydraulic cylinder;
- viscous friction forces resulting from fluid flow through pipelines;
- static force of gravity and dynamic forces arising in the hydraulic cylinder from the side of the piston and piston rod chamber due to the pressure of the working fluid.

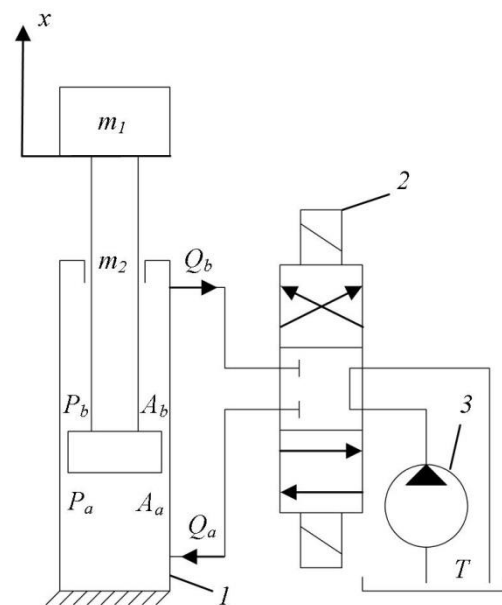


Figure 1. The design scheme of the hydraulic system.

In Fig. 1 the following designations are accepted: 1 – double acting hydraulic cylinder; 2 – hydraulic distributor; 3 – hydraulic pump; T – tank with working fluid, x – coordinate movement of the cylinder piston rod, m ; m_1 – mass of cargo, kg ; m_2 – mass of piston rod, kg ; P_a and P_b – the pressure of the working fluid in the piston and piston rod chambers of the hydraulic cylinder, respectively, Pa ; A_a and A_b – respectively, the area of the piston chamber and the piston rod chamber of the hydraulic cylinder, m^2 ; Q_a and Q_b – the flow rate of fluid for the piston and piston rod chamber of the cylinder, respectively, m^3/s .

In accordance with accepted assumptions, operating forces and design scheme Fig. 1, according to D'Alembert principle the differential equation of motion of the piston

rod of the hydraulic cylinder will have the following form:

$$m \frac{d^2 x}{dt^2} + B \frac{dx}{dt} = P_a \cdot A_a - P_b \cdot A_b - m \cdot g, \quad (1)$$

where: m – reduced mass of moving parts and working fluid, kg ; x – displacement of the piston rod of the hydraulic cylinder, m ; B – coefficient of damping force, associated with the presence of viscous and dry friction forces, $\frac{H}{m/s}$; g – free fall acceleration, m/s^2 .

The change in pressure in the cavities of the cylinder is determined from the equations of continuity of the flow, taking into account the compression of the working fluid in the chambers of the cylinder and the elements of the pipeline. Accordingly, we will have the following dependencies:

-for piston chamber:

$$\frac{dP_a}{dt} = \frac{E}{V_a} \cdot \left(Q_a - A_a \cdot \frac{dx}{dt} \right); \quad (2)$$

$$V_a = V_{\min} + A_a \cdot x, \quad (3)$$

-for piston rod camera:

$$\frac{dP_b}{dt} = \frac{E}{V_b} \cdot \left(-Q_b - A_b \cdot \frac{dx}{dt} \right); \quad (4)$$

$$V_b = V_{\max} - A_b \cdot x, \quad (5)$$

where: E – bulk modulus of elasticity, Pa ; V_a and V_b – according to the initial volume of working fluid in the piston and rod chambers of the hydraulic cylinder with adjacent to pipelines, m^3 ; $V_{\min}$ – the volume of working fluid in the piston chamber and adjacent to pipelines at the zero position of the piston ($x=0$), m^3 ; $V_{\max}$ – the volume of working fluid in the rod chamber and adjacent to pipelines at zero piston position ($x=0$), m^3 ; Q_a and Q_b – respectively the fluid flow in the piston and piston rod chambers of the hydraulic cylinder, m^3/s .

Bulk modulus of elasticity (Kostyunichev *et al.* 2011):

$$E = \frac{E_f}{1 + \frac{d}{\delta} \cdot \frac{E_f}{E_s}}, \quad (6)$$

where: E_f and E_s – respectively, the bulk modulus of the working fluid and the pipeline material, Pa ; d and δ – respectively the diameter and thickness of the wall of the hydraulic line, m .

Hydraulic distributor, the principle of which is shown in Fig. 2, is considered as a control equipment for moving a double-acting hydraulic cylinder with the following functions:

-redistribution of flows of the working fluid in the hydraulic system, respectively, the supply to the piston or piston rod chamber of the cylinder and the removal into the tank of the working fluid from the piston and piston rod chambers of the cylinder;

-change of speed of movement of a piston rod of the hydraulic cylinder depending on movement of the hydraulic slide valve relative to the sleeve and formation of a cross-section between the edges of the slide valve and the inlet and outlet channels of the housing (Fig. 2).

Accordingly, the fluid consumption will be as follows:

-for supplying fluid into the piston chamber:

$$Q_a = \mu \cdot f_a \sqrt{\frac{2}{\rho} |P_n - P_a|}; \quad (7)$$

-for supplying fluid into the piston rod chamber:

$$Q_b = \mu \cdot f_b \sqrt{\frac{2}{\rho} |P_b - P_d|}, \quad (8)$$

where: μ – the coefficient of consumption of the working fluid; f_a and f_b the cross-sectional area, m^2 ; ρ – specific gravity of the working fluid, kg/m^3 ; P_n – the pressure of the working fluid is created by a hydraulic pump, Pa ; P_d – the pressure of the working fluid in the drain line, Pa .

To determine the magnitude of the damping force factor, we present it as the sum of two coefficients:

$$B = F_h + F, \quad (9)$$

where: F_h – force of hydraulic resistance, N ; F – the force of semi-dry friction in the seals and guide sleeves according to the piston rod and piston of the hydraulic cylinder, N .

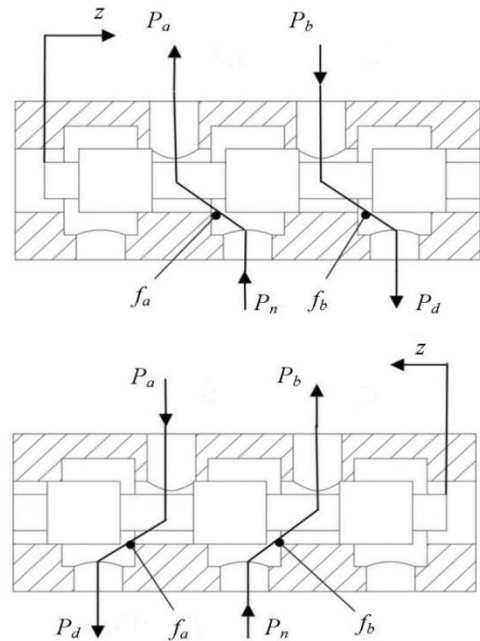


Figure 2. The principle of operation of the hydraulic distributor.

The force of hydraulic resistance can be found from the following dependence (Samusenکو, 1981):

$$F_z = \frac{4 \cdot \frac{\lambda \cdot Re}{2} \cdot \mu_d \cdot l_1 \cdot A_b}{\pi \cdot d^4}, \quad (10)$$

where: μ_d – dynamic viscosity of the working fluid, $Pa \cdot s$; l_1 – length of pipeline, m ; π – mathematical constant; λ – coefficient of hydraulic friction equal to:
- for laminar mode:

$$\lambda = \frac{37.5 \cdot \frac{dx}{dt}}{V_f \cdot d}, \quad (11)$$

where: V_f – the speed of the fluid in the pipeline;

$$V_f = \frac{4 \cdot A_b}{\pi \cdot d} \cdot \frac{dx}{dt}; \quad (12)$$

- for transient turbulent mode ($2300 < Re < 8 \cdot 10^4$):
 $\lambda = 0,3164 \cdot Re^{-0,25}$. (13)

Determined the forces of semi-dry friction in the seals and guide sleeves of the rod and piston of the hydraulic cylinder [Kondakov *et al.* 1994]:

$$F = F_1 + F_2 + F_3, \quad (14)$$

where: F_1 – the force of the semi-dry friction of the piston rod, N ; F_2 – the force of the semi-dry friction of the piston, N ; F_3 – resistance force of fluid flow from the chamber to the tank, N .

$$F_1 = \mu_1 \cdot \pi \cdot d_2 \cdot b(n \cdot p_k + P_b), \quad (15)$$

where: μ_1 – coefficient of friction of sealing cuffs on work surfaces; d_2 – diameter of the piston rod, m ; b – the width of the contact of the sealing rings, m ; n – the number of sealing rings; p_k – the initial specific pressure of the ring on the work surface, Pa .

$$F_2 = \mu_1 \cdot \pi \cdot d_3 \cdot b(n \cdot p_k + P_a), \quad (16)$$

where: d_3 – diameter of the cylinder, m .

$$F_3 = P_d \left(\frac{\pi \cdot d_3^2}{4} - \frac{\pi \cdot d_2^2}{4} \right). \quad (17)$$

Thus, the dependencies (1)–(17) allow us to describe the dynamics of the double-acting hydraulic cylinder, and the dependencies (1)–(8) are the basic equations of the mathematical model of the hydraulic system.

To solve the mathematical model and studies of the dynamics of displacement of the rod hydraulic cylinder with cargo, accepted, what a change in the cross-section f_a and f_b in hydraulic distributor varies by such dependencies:

a) Linear dependence:

- for piston chamber,

$$f_a \max \cdot \frac{t}{\Delta t}; \quad (18)$$

- for rod chamber,

$$f_b \max \cdot \frac{t}{\Delta t}; \quad (19)$$

b) Parabolic dependence:

- for piston chamber,

$$f_a \max \cdot \frac{t^2}{\Delta t^2}; \quad (20)$$

- for rod chamber,

$$f_b \max \cdot \frac{t^2}{\Delta t^2}; \quad (21)$$

c) S-shaped dependence:

- for piston chamber,

$$f_a \max \cdot \left(\frac{6-8 \cdot t}{\Delta t} + \frac{3 \cdot t^2}{\Delta t^2} \right) \cdot \frac{t^2}{\Delta t^2}; \quad (22)$$

- for rod chamber,

$$f_b \max \cdot \left(\frac{6-8 \cdot t}{\Delta t} + \frac{3 \cdot t^2}{\Delta t^2} \right) \cdot \frac{t^2}{\Delta t^2}; \quad (23)$$

where: t – time of rod movement, s ; Δt – time of moving the slide valve, s .

Reduce mass determine the mass taking into account the change in the weight of the working fluid in the piston rod chamber. Then we get:

$$m = m_1 + m_2 + V_b \cdot \rho. \quad (24)$$

To solve the mathematical model, we take the following initial parameters:

$m_1 = 15000 \text{ kg}$; $m_2 = 41 \text{ kg}$; $A_a = 0,01226565 \text{ m}^2$;

$A_b = 0,007241625 \text{ m}^2$; $P_n = 20 \cdot 10^6 \text{ Pa}$;

$P_d = 1 \cdot 10^6 \text{ Pa}$; $E_f = 1,3 \cdot 10^8 \text{ Pa}$; $E_s = 2 \cdot 10^{11} \text{ Pa}$;

$d = 1,2 \cdot 10^{-2} \text{ m}^2$; $\delta = 2 \cdot 10^{-3} \text{ m}^2$; $\rho = 850 \text{ kg/m}^3$;

$l_1 = 0,5 \text{ m}$; $d_2 = 0,08 \text{ m}$; $d_3 = 0,125 \text{ m}$;

$f_a \max = f_b \max = 1,45 \cdot 10^{-5} \text{ m}^2$; $t = 2 \text{ s}$;

$\Delta t = 0,25 \text{ s}$.

Initial conditions:

$$x[0] = 0, \quad \frac{dx}{dt}[0] = 0, \quad P_b[0] = \frac{V_b \cdot \rho \cdot g}{A_b}, \quad P_a[0] = \frac{m \cdot g}{A_a}.$$

By solving mathematical model by numerical methods according to accepted initial parameters and initial data, we obtained graphical dependences of speed, acceleration, and changes of pressure in the piston and rod chambers during the transient process of movement of the rod of a hydraulic cylinder of double action (Fig. 3 – Fig. 6). Analyzing and comparing the obtained graphical dependences of the speed of movement of the rod of the hydraulic cylinder (Fig. 3) in accordance with the dependence of the change of the area of the hydraulic distributor, we can note the following:

- by linear dependence of the change in the cross-sectional area of the hydraulic distributor Fig. 3a, the acceleration of the hydraulic cylinder piston rod with the load continues 0,25s, which corresponds to the time of movement of the slide valve $\Delta t = 0,25s$.

During acceleration, dynamic loads occur that correspond to fluctuations in the working fluid pressure (Fig. 5 a, in the interval of time $t = 0 - 0,25s$) and acceleration (Fig. 4a, in the interval of time $t = 0 - 0,25s$). Further the speed of movement of the rod has a oscillatory character, which disappears within 2c. The maximum speed value is equal to 0,108 m/s, and the minimum 0,095m/s. Steady motion speed is 0,1m/s;

- by parabolic dependence, changes in the cross-sectional area of the hydraulic distributor (Fig. 3 b) speed up of the piston rod of the hydraulic cylinder with the load continues 0,25 s, which corresponds to the time of movement of the valve spool $\Delta t = 0,25s$. Speed up of the hydraulic cylinder piston rod is smooth without dynamic loads, which corresponds to the schedule of change of pressure of the working fluid (Fig. 5 b, in the interval of time $t = 0 - 0,25s$) and acceleration (Fig. 4 b, in the interval of time $t = 0 - 0,25s$). Further the speed of movement of the rod has a oscillatory character, which disappears within 2s. The maximum speed value is equal to 0,11m/s, and the minimum 0,093m/s. Steady motion speed is 0,1m/s;

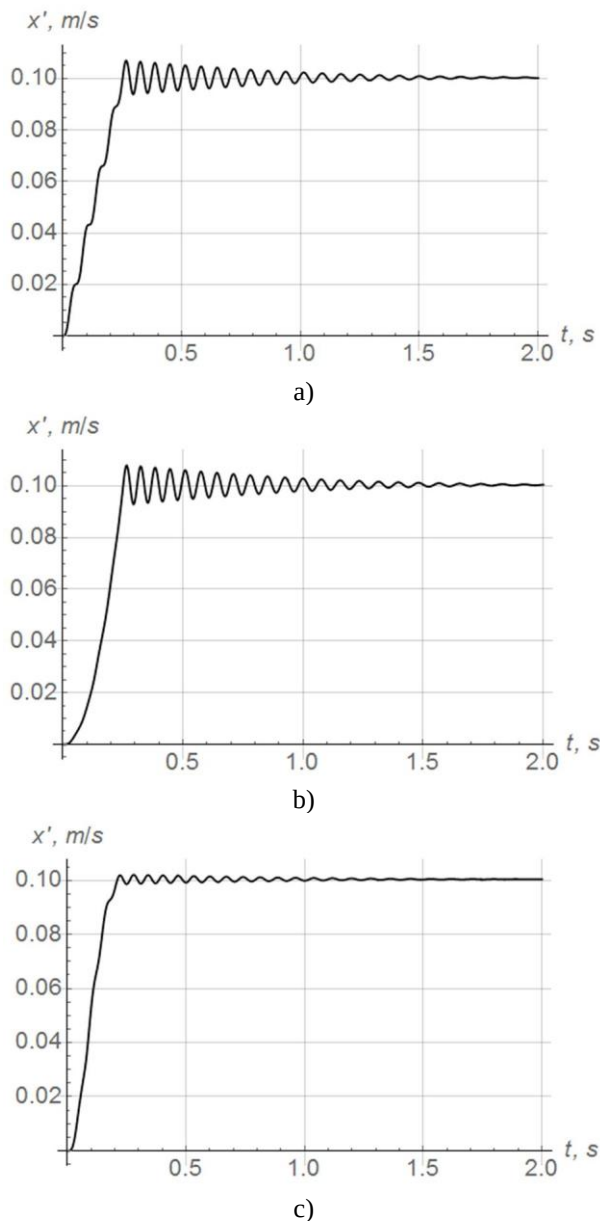


Figure 3. Graph of the functional dependence of the speed of movement of the rod of hydraulic cylinder: a) linear dependence; b) parabolic dependence; c) S-shaped dependence.

- by S-shaped dependence changes in the cross-sectional area of the hydraulic distributor Fig. 3 c) speed up of the rod of the hydraulic cylinder with the load continues 0,2s, which is less on 0,05s, than dependencies (18-21). During speed up slight dynamic loads occur that correspond to fluctuations in the working fluid pressure (Fig. 5 c, in the interval of time $t = 0 - 0,2s$) and acceleration (Fig. 4 c, in the interval of time $t = 0 - 0,2s$). Further the speed of movement of the rod has a oscillatory character, which disappears within 1,3c. Steady motion speed is 0,1m/s.

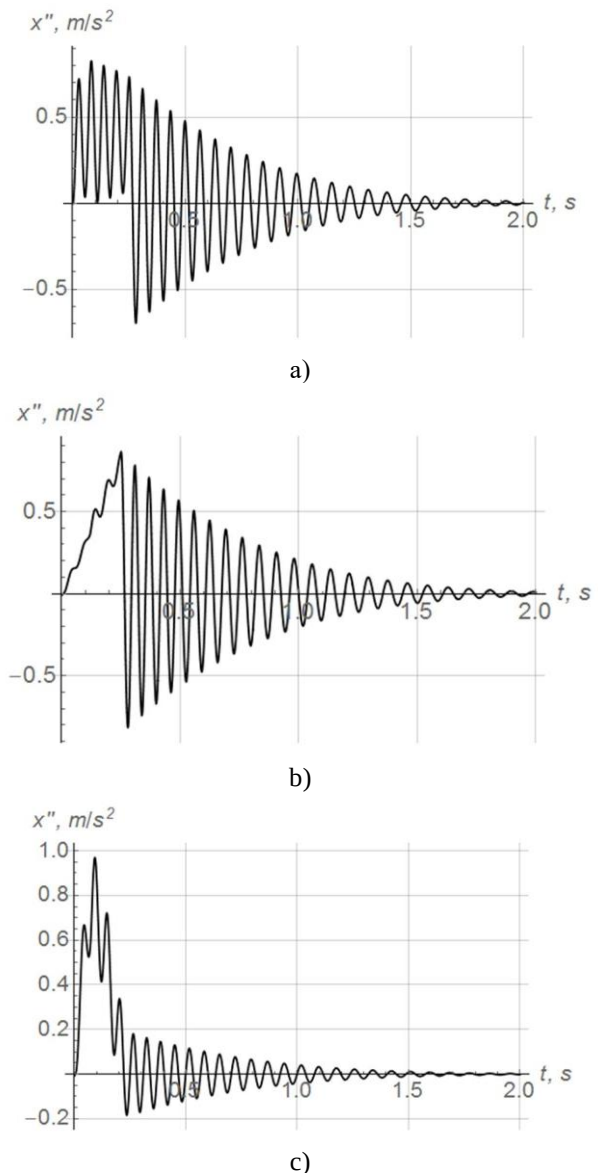


Figure 4. Graph of the functional dependence of the acceleration of the rod of hydraulic cylinder: a) linear dependence; b) parabolic dependence; c) S-shaped dependence.

Analysing and comparing the obtained graphical dependencies of the acceleration of rod of the hydraulic cylinder (Fig. 4) in accordance with the dependence of the change of the area of the hydraulic distributor, we can note the following:

- by linear dependence of the change in the cross-sectional area of the hydraulic distributor (Fig. 4 a): during the speed up of rod of the hydraulic cylinder ($t = 0 - 0,25s$) acceleration is accompanied by considerable fluctuations and acquires a maximum value of $0,82m/s^2$. When further moved piston rod of the hydraulic cylinder acceleration takes the form vibrational loads occur of alternating oscillation character;

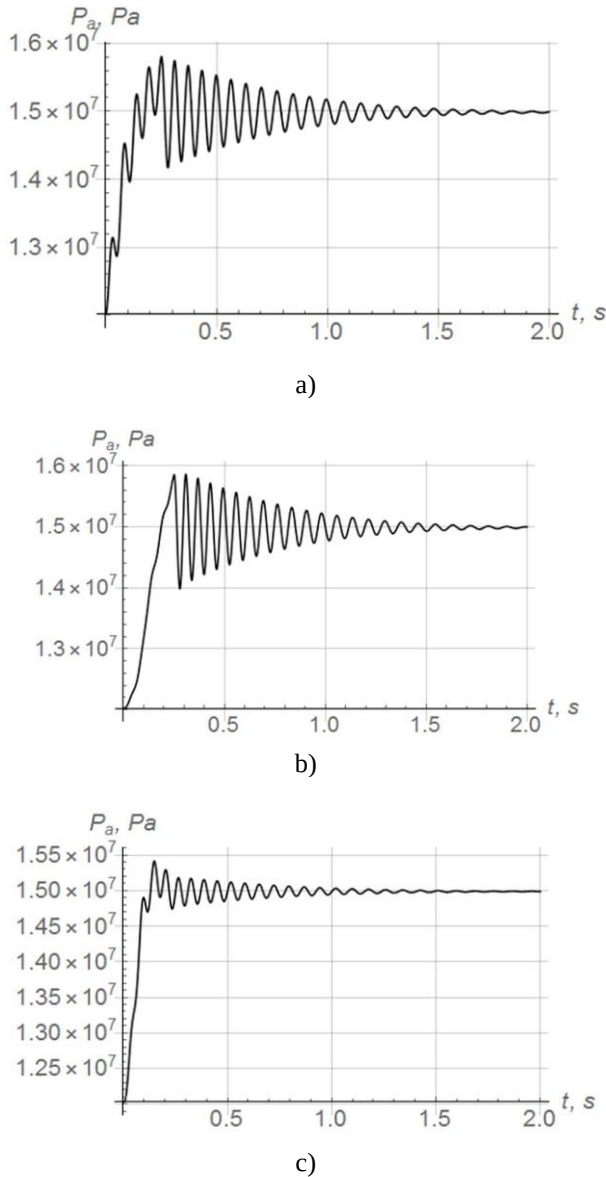


Figure 5. Graph of the functional dependence of the changes in pressure in the piston chamber of the hydraulic cylinder: a) linear dependence; b) parabolic dependence; c) S-shaped dependence.

- by parabolic dependence, changes in the cross-sectional area of the hydraulic distributor (Fig. 4 b): during the speed up piston rod of the hydraulic cylinder ($t = 0 - 0,25s$) acceleration occurs evenly with slight oscillations. The maximum acceleration value becomes in moment when $t = 0,25s$, $f_a = f_a \max$, $f_b = f_b \max$

and is equal to $0,88m/s^2$. When further moved piston rod of the hydraulic cylinder acceleration takes the form vibrational loads occur of alternating oscillation character;

- by S-shaped dependence, changes in the cross-sectional area of the hydraulic distributor (Fig. 4 c): on the time speed up ($t = 0 - 0,25s$) acceleration occurs evenly with slight oscillations. The maximum acceleration value becomes in moment when $t = 0,1s$ and is equal to $0,98m/s^2$. In the moment of time $t = 0,25s$, the acceleration of rod of the hydraulic cylinder is $0,195m/s^2$. When further moved rod of the hydraulic cylinder acceleration takes the form vibrational loads occur of alternating oscillation character.

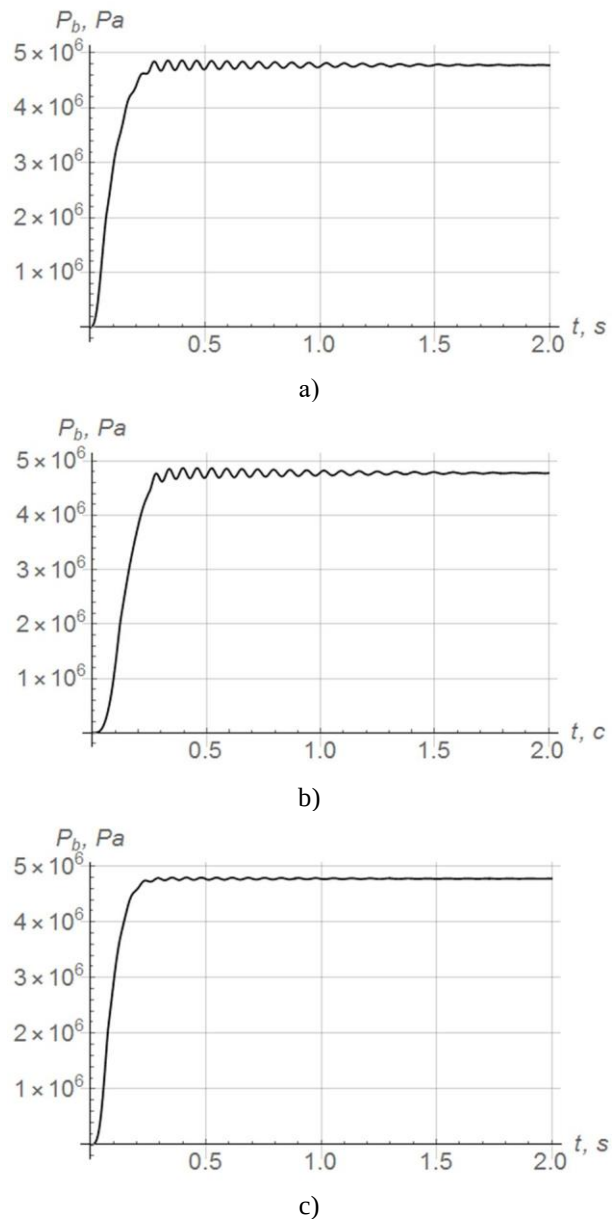


Figure 6. Graph of the functional dependence of the changes in pressure in the piston rod chamber of the hydraulic cylinder: a) linear dependence; b) parabolic dependence; c) S-shaped dependence.

Analysing and comparing the obtained graphical dependencies of the changes in pressure in the piston chamber hydraulic cylinder (Fig. 5) in accordance with the dependence of the change of the area of the hydraulic distributor, we can note the following:

-by linear dependence of the change in the cross-sectional area of the hydraulic distributor (Fig. 5 a): during the speed up of rod of the hydraulic cylinder ($t = 0 - 0,25s$) significant pressure fluctuations occur, which adversely affect the mode of motion and the acceleration of rod of the hydraulic cylinder. The maximum pressure value is $1,58 \cdot 10^7 Pa$, in the moment of time $t = 0,25s$. Further movement of the piston rod of the hydraulic cylinder accompanied by slight fluctuations in pressure. This is due to the inertial component of the moving masses, the compressibility of the working fluid and the elasticity of the elements of the hydraulic system. Oscillation damping occurs within 2s. The pressure in the piston chamber during steady motion is $1,5 \cdot 10^7 Pa$;

-by parabolic dependence of the change in the cross-sectional area of the hydraulic distributor (Fig. 5 b): during the speed up of rod of the hydraulic cylinder ($t = 0 - 0,25s$) pressure fluctuations are absent. The maximum pressure value is $1,58 \cdot 10^7 Pa$, in the moment of time $t = 0,25s$. Oscillation damping occurs within 2s. The pressure in the piston chamber during steady motion is $1,5 \cdot 10^7 Pa$;

-by S-shaped dependence of the change in the cross-sectional area of the hydraulic distributor (Fig. 5 c): during the speed up of rod of the hydraulic cylinder ($t = 0 - 0,1s$) pressure fluctuations are absent. In the moment of time $t = 0,1 - 0,25s$ slight fluctuations in pressure occur, the maximum value of which is $1,54 \cdot 10^7 Pa$. Oscillation damping occurs within 1,3s. The pressure in the piston chamber during steady motion is $1,5 \cdot 10^7 Pa$.

Conclusions

As a result of the research, a mathematical model was constructed taking into account the compressibility of the working fluid, the flexibility of the hydraulic drive elements and the influence of the forces of viscous and semi-dry friction in the hydraulic drive elements. A dynamic analysis of the movement of the rod of hydraulic cylinder a double-acting was carried out for different dependences of the change in the passage area in the hydraulic distributor. Dynamic loads in the elements of the hydraulic system were obtained by analysing which, it is evident that it is desirable to apply a change in the cross-sectional area according to S-dependence. Because at this dependence dynamic loads a minimal.

The proposed mathematical model makes it possible to determine the actual dynamic loads in the hydraulic system under transient modes of motion. The

mathematical model obtained can be used at the design stage of hydraulic systems of hoisting and other machines.

References

- Anisimov A. V., Kondrashev V. L., Lihoded K. A., Shoshiashvili M. E. 2012. Dinamika gidrosistem. Novocheboksarsk. 131 p.
- Bashta T. M. 1971. Mashinostroitel'naya gidravlika. Moscow. 672 p.
- Detiček E. 2011. An intelligent electro-hydraulic servo drive positioning. *Journal of Mechanical Engineering*. Vol. 57(2011). Issue 5. P. 394-404.
- Kondakov L. A., Golubev A. I., Ovander V. B., Gordeev V. V., Furmanov B. A., Karmugin B. V. 1994. Uplotneniya i uplotnitelnaya tekhnika. Moscow. 448 p.
- Kostyunichev D. N., Nikitaev I. V., Cvetkova E. V. 2011. Osnovi rascheta obemnogo gidroprivoda. Nizhny Novgorod: 96 p.
- Ming Xu, Jing Ni, Guojin Chen. 2014. Dynamic simulation of variable-speed valve controlled-motor drive system with a power-assisted device. *Journal of Mechanical Engineering*. Vol. 60(2014). Issue 9. P. 581-591.
- Mintsa H. A. 2012. Feedback linearization-based position control of an electrohydraulic servo with supply pressure uncertainty. *IEEE Transaction on Control System Technology*. No 4. P. 1092-1099.
- Mishuk D. O. 2016. Doslidzhennia dynamichnoi modeli hidrosylindra obiemnogo hidravlichnogo pryvodu. Girnichy, budivelni, dorozhni i meliorativni mashini, Kyiv. No 2. P. 74-81. (in Ukraine).
- Pelevin L. Ye., Mishuk D. O., Rashkivskij V. P., Gorbatiuk Ye. V., Arzhayev G. O., Krasnikov V. F. 2015. Hidravlika, hidromashyny ta Hidropnevmoavtomatyka. Kyiv: 340 p. (in Ukraine).
- Popov D. N. 1987. Dinamika i regulirovanie gidropnevmaticheskikh sistem. Moscow: 424 p.
- Samusenko M. F. 1981. Dinamika gidravlicheskikh mekhanizmov podema gruzopodemnogo oborudovaniya. Moscow. 60 p.
- Zeizin V. G. 2011. Dinamika i regulirovanie gidropnevmo sistem. Chelyabinsk. 146 p.
- Zhdanov A. V. 2016. Matematicheskaya model raspredelitelya pozicionnogo gidroprivoda stroitelno-dorozhnykh mashin. Omskij nauchnyj vestnik. Omsk. No 4. P. 41-44.
- Zhilevich M. I., Ermilov S. V., Kishkevich P. N., Bigel E. N. 2013. Dinamicheskij raschet gidravlicheskogo raspredelitelya. Mashinostroenie i mashinovedenie. *Vesnik GGTU im. P.O. Sukhogo*. Gomel. No 2. P. 11-16.